UNCLASSIFIED

GIMRADA Research Note No. 9

GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS

By K. Bertil P. Hallert

18 September 1962

U.S. ARMY
GEODESY, INTELLIGENCE AND MAPPING
RESEARCH AND DEVELOPMENT AGENCY
CORPS OF ENGINEERS

FORT BELVOIR VA

Correction Sheet
GIMRADA Research Note No. 9
"Geometrical Quality of Lunar Mapping
By Photogrammetric Methods"

page 4	line 16 from bottom	delete the comma sign after identically
page 4	line 7 from bottom	change $d\eta_o$ into dh_o
page 5	line 6 from top	change Y_1 into Y_2
page 5	line 7 from top	change Y_2 into Y_3
page 5	line 9 from top	change dh into [dh]
page 22	line 2 from bottom	read (x_r), (y_r), (c) are:
page 23	Figure 2	change κ_{II} into κ_{III}
page 27	expression 15	change the expression x" into x' in the denominator

U. S. ARMY ENGINEER
GEODESY, INTELLIGENCE AND MAPPING RESEARCH AND DEVELOPMENT AGENCY
FORT BELVOIR, VIRGINIA

Research Note No. 9

GEOMETRICAL QUALITY OF LUNAR MAPPING

BY PHOTOGRAMMETRIC METHODS

Task 8T35-12-001-02

18 September 1962

Distributed by

The Director
U. S. Army Engineer
Geodesy, Intelligence and Mapping Research and Development Agency

Prepared by

K. Bertil P. Hallert
Research and Analysis Division
U. S. Army Engineer
Geodesy, Intelligence and Mapping Research and Development Agency
Fort Belvoir, Virginia

CONTENTS

SUMMARY

For the determination of coordinates and elevation on the moon from stereoscopic photographs, ordinary photogrammetric methods are assumed to be used. The highest geometrical quality of the intersected coordinates is to be expected from numerical computations according to the formulas for convergent photogrammetry, to be found in any of the more comprehensive textbooks on photogrammetry.

The basic errors of the procedure emanate from the image coordinates in the photographs and are propagated through the formulas used according to the laws of error propagation. Regular (systematic) errors of the image coordinates are assumed to be well determined from the calibration of the camera, and an estimation of the irregular errors should always be done in the calibration according to the method of least squares and should be expressed as a standard error of unit weight.

An estimation of this basic factor is also possible from discrepancies in the condition that corresponding reconstructed rays from the two photographs shall intersect in space. An adjustment of such discrepancies according to the method of least squares will give the standard error of unit weight of the observations behind the discrepancies in the intersection condition (in photogrammetry, usually denoted as y-parallaxes). This standard error of unit weight is closely related to the standard error of unit weight of the image coordinates and can consequently be used for an estimation of this basic factor for each individual pair of photographs.

From this standard error of unit weight, the standard errors of all intersected point coordinates can be determined according to the laws of error propagation. In order to express these standard errors in a local coordinate system, the principle of compensation is applied to those intersected points which are to be used for the definition of the local coordinate system. General expressions for the geometrical quality to be expected in the planimetric coordinates and in the elevations have been derived and express this quality in terms of standard errors as functions of the basic standard errors of unit weight of the image coordinates.

So far, no determination of this basic factor has been made or reported. The value used in the examples, 0.01 mm, is estimated with the aid of some preliminary measurements for the determination of the <u>precision</u> of image coordinate measurements in moon photographs in a <u>comparator</u>, i.e., from repeated and replicated measurements only.

It should also be emphasized that certain approximations, probably unimportant in comparison with other errors, have been used, primarily in neglecting the angle of convergency in the basic error propagation expression. Further, the coordinates and the elevations are referred to a plane coordinate system. The transformation into selenographic coordinates is a separate operation, not treated here.

If the interior orientation of the photographs is determined according to ordinary photogrammetric methods, the angle of convergency (the libration angle) can be determined, in principle, through the adjustment procedure. The geometrical quality (standard error) of this angle is a simple function of the standard error of unit weight of the image coordinates too. A low quality, however, must be expected, because of the narrow bundles of rays.

The photogrammetric procedure for the determination of the relative orientation and for the estimation of the geometrical quality of this operation can be directly applied to moon photographs. If the interior orientation of the moon photographs is not determined but the stereoscopic photographs are taken with the same camera, it may be possible to determine the angle of convergency, provided that there are details in the plate magazine or in the frame of the supporting plane which can be identified from plate to plate.

The photogrammetric procedure can be studied in available textbooks together with the basic theory of errors to be applied (see for instance references [2], [3], and [4] and the appendix).

Assuming various scales, planimetric and elevation requirements for lunar maps and photographs, it is possible to formulate a method for determining the dependency of the geometrical quality. Under the conditions chosen, it is shown that the lunar instrumentation required is unrealistic. Estimates of the standard errors from stereophotogrammetric measurement in photographs taken with a telescope of 150-foot effective focal length are tabulated for different values of the standard error of unit weight (s_0) of image coordinates and parallaxes on the scale of the images. In addition, estimations of the relative geometrical quality from available satellite and balloon photography are given.

It is concluded that a comprehensive evaluation of the requirements and the necessary geometric quality is warranted for all planned lunar mapping and photography.

GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS

I. INTRODUCTION

All photogrammetric methods are founded upon the concept of image coordinates; therefore, the geometrical quality of the final results is primarily dependent upon the geometrical quality of the calibration of the camera and of the photographs. Cameras for moon mapping purposes, as well as all measurements, must therefore be carefully calibrated under operational conditions. In particular, the standard error of unit weight of the image coordinates must be determined if realistic estimations of the geometrical quality to be expected in the final results shall be made possible. So far, no determination of the basic standard errors of unit weight of the image coordinates has been made or reported.

Stereophotogrammetric methods are assumed to be used for mapping the moon from stereoscopic photography obtained with the aid of the librations. From measurements of image coordinates and coordinate differences (parallaxes) in the photographs, ordinary photogrammetric methods can be used for the determination of the three coordinates of the intersection of the reconstructed rays from the photographs. The coordinates are to be determined primarily according to the convergent case of photography and are first obtained in a system, the origin of which is located in one of the camera stations, usually the left one (see Appendix). The numerical procedure for the computation of the coordinates is more accurate than procedures in the ordinary photogrammetric plotting instruments.

Since the coordinates are determined from measured data via formula systems, it is possible to determine the geometrical quality to be expected through a study of the error propagation from the basic measurements according to usual procedures. The geometrical quality of the basic measurements, primarily of the image coordinates, must be determined or estimated in some way as a standard error of unit weight. This fundamental quality can be determined in connection with calibration of the camera, but this calibration must be made under operational conditions. Another possibility is to use the condition that reconstructed pairs of rays from stereoscopic photographs shall intersect in space. From discrepancies in these conditions and adjustments according to least squares, using the elements of the relative orientation of the bundles of rays and other possible "regular" sources of parallaxes as parameters, a statistical value of the residual discrepancies after the adjustment can be determined. This is essentially the same procedure for determining the standard error of unit weight of y-parallaxes in aerial photogrammetry.

Since such parallaxes are direct functions of image coordinates, the standard error of the image coordinates can be estimated from the results of the adjustment of the intersections of the reconstructed pairs of rays. This method of determining the basic accuracy of image coordinate measurements can be applied to arbitrary pairs of stereoscopic survey photographs to be used for measuring purposes and is very convenient, since the condition is furnished by the photographs themselves and no other conditions in terms of given data are required.[1] It should be noted, however, that there may be correlation effects between the two bundles of rays which may compensate important regular (systematic) errors of the individual rays and, therefore, the relation between the standard errors of unit weight of the image coördinates and of the parallaxes may be closer to 1:1 than to $1:\sqrt{2}$. In aerial photogrammetry, the relation 1:1 has been used with acceptable results. Consequently, if the condition of intersection of rays (y-parallax condition) is used for the determination of the basic geometrical quality of the image coordinate measurements, it is advisable to use the obtained standard error of unit weight as an estimation of the standard error of unit weight of the basic image coordinates too. Measurements and computations for the determination of this basic factor are comparatively simple (see references [2] and [3]).[2] Stereoscopic measuring methods should always be used for the purpose of reliable identification in the two photographs.

Through differential formulas of the analytical expressions used for the computation of the three coordinates of intersected points on the moon's surface, the influence of the errors of the measurements upon the coordinates can be studied. If the errors of the measurements are determined as indicated above, i.e., as statistical expressions (standard errors of unit weight), the standard errors of the intersected coordinates can be computed according to the special law of error propagation (reference [2]). It is then assumed that the errors of the basic measurements are normally distributed on a reasonable level, usually 5 percent. Tests of the statistical distribution of the residuals in a sufficient number of redundant observations in connection with the determination of the basic geometrical quality are therefore always advisable. If, for some reason, the basic measurements are expected to be of different quality, the corresponding (a priori) weights can be estimated and introduced in the calculations. There is a clear correlation between the resolving power and the standard error of unit weight. The

[1] The elements of the interior orientation must be known in survey photographs and also the accuracy of the determination.

[2] See also the Appendix. Additional parameters, for instance possible "vertical" libration, can be introduced in the adjustment procedure.

resolving power decreases and the standard error of unit weight increases with the distance from the center. Therefore, it is most probable that points on the moon, located close to the limbs, will be measured with less geometrical quality than those in the center of the moon and that, consequently, a weight variation could be estimated. Weights can also be introduced according to the intersection geometry. The size and directions of the shadows are also of great importance for the identification.

The errors of the intersected points, determined in the indicated manner, refer to the origin of the coordinate system, temporarily located in the left camera station. For the determination of the geometrical quality of the mapping procedure, this type of errors is of limited interest since the map primarily shall show the local geometrical (topographical) relations on the moon. Therefore, the local geometrical quality must be of primary importance in connection with the mapping. The coordinates and the elevations shall, in other words, be referred to a local trigonometric network or leveling system on the moon. Therefore, temporary coordinate systems are created by the necessary number of intersected points, two in planimetry and three in elevation. This means that all intersected points are transformed numerically into a coordinate system defined by the seven mentioned parameters, which are represented by suitably located and well-defined intersected points. The procedure is identical with absolute orientation of a model in photogrammetry. The most important task is now to determine the geometrical quality to be expected in the coordinates x and y and the elevations h in this local system. For this purpose, the principle of compensation should be used. The final transformation into selenographic coordinates is not considered here.

In planning the lunar mapping project, telescopes, cameras, plotting methods, instruments, operators, and control density must all be chosen so as to allow the coordinates of the photogrammetric model to be determined with a root mean square value of the standard errors smaller or equal to the required values. General formulas have been indicated for the standard errors in planimetry and elevation. The standard errors should be expressed as functions of the standard errors of the fundamental operations, and attention should be paid to the balancing effects of the operations and of the control points. The number and location of the control points will influence the accuracy to be expected. Therefore, the expressions for the accuracy have to be specialized for control-point combinations.

No attention has been paid to the interpretation quality required of the photography for the selection and interpretation of the map details. For a detailed study of this problem, the requirements on the map details must be specified and the interpretative qualities of the camera and photographs must be investigated. The resolving power of the lens-negative-diapositive-plotting instrument-combination must be defined and tested.

II. APPLICATION OF THE PRINCIPLE OF COMPENSATION TO INVESTIGATIONS OF THE GEOMETRICAL QUALITY OF LUNAR MAPPING WITH PHOTOGRAMMETRIC METHODS.

1. Determination of the Accuracy to be Expected in Elevation Differences on the Moon. An assumption is made that a photogrammetric method has been applied for the determination of elevation differences on the moon. The z-coordinates (in the direction orthogonal to the base) have been obtained from the intersection of reconstructed rays. An estimation of the accuracy (standard errors) of the intersected points has been made according to the used numerical formulas and as a function of the standard error of the image coordinates. This standard error has been estimated in connection with the calibration of the camera and/or from y-parallax determinations and adjustments according to the method of least squares.

A certain variation of the accuracy of intersected points is to be expected, due to the variations in the photographic quality and identification. The geometrical conditions will also have some influence upon the accuracy. The variations of the accuracy can be expressed in terms of weights to be assigned to the z-coordinate. In the following derivation, equal weights are assumed for the sake of simplicity, but identically, the same procedure can be used for weighted observations and intersections.

Three elevation points are selected which define the reference plane. The elevation differences between the points can be assumed as known, but with a degree of uncertainty, since the points were determined from other measurements. The errors are to be of differential order of magnitude.

The intersected point group is transformed to the elevation system of the reference plane. The errors of the position of the reference plane will be entirely compensated in the three points by the transformation elements which are one translation, $d\eta_o$, and two rotations, $d\eta$ and $d\xi$. It is assumed that the coordinates x and y of the three points are translated to a coordinate system, the origin of which is located at the center of gravity of the three points. The differential relation between elevation errors of points in the plane and the mentioned parameters is of the form:

$$dh = Xd\eta + Yd\xi + dh_o$$

where X and Y are the gravity coordinates.

Elevation errors dh_1, dh_2, and dh_3 in the three points used for the coordinate transformation are related to the parameters dh_o, $d\eta$, and $d\xi$ through the equations (written as correction equations):

$$dh_1 = -X_1 d\eta - Y_1 d\xi - dh_o$$

$$dh_2 = -X_2 d\eta - Y_1 d\xi - dh_o$$

$$dh_3 = -X_3 d\eta - Y_2 d\xi - dh_o$$

This system can be solved as follows:

$$dh_o = -\frac{dh}{3}$$

$$d\eta = \frac{[XY]\,[Ydh] - [YY]\,[Xdh]}{[XX]\,[YY] - [XY]^2}$$

$$d\xi = \frac{[XY]\,[Xdh] - [XX]\,[Ydh]}{[XX]\,[YY] - [XY]^2}$$

Corrections can now be computed to the elevations of arbitrary points with the aid of the original expression. The geometrical quality of the corrections can be computed from the same expression in applying the general law of error propagation. For this purpose, the weight and correlation number of the parameters from the solution above have to be determined. According to the definition of such numbers, we find:

$$Q_{h_o h_o} = \frac{1}{3}$$

$$Q_{\eta\eta} = \frac{[YY]}{[XX]\,[YY] - [XY]^2}$$

$$Q_{\xi\xi} = \frac{[XX]}{[XX]\,[YY] - [XY]^2}$$

$$Q_{\eta\xi} = \frac{[XY]}{[XX]\,[YY] - [XY]^2}$$

For points which are located in the vicinity of the reference plane, the final elevation after the correction can be expressed by the formula:

$$h_{final} = h_{prelimin.} + dh_o + Xd\eta + Yd\xi$$

Assuming the same geometrical quality (standard error) in the measurements of $h_{prelimin.}$ as of the elevations in the points 1, 2 and 3 from which the corrections are to be computed, we find the weight number of the final elevation of a point X_i, Y_i in the neighborhood of the reference plane:

$$Q_{h_i h_i} = 1 + \frac{1}{3} + \frac{[YY]\,X_i^2 + [XX]\,Y_i^2 - 2\,[XY]\,X_i\,Y_i}{[XX]\,[YY] - [XY]^2}$$

The standard error of the elevation is then found from

$$s_{h_i} = s_{oh}\sqrt{Q_{h_i h_i}}$$

s_{oh} is the standard error of unit weight of the elevation measurements from the intersection procedure. This error is primarily a parallax error which can be determined from the condition that rays shall intersect in space and then transformed to an elevation error with the aid of the distance-base ratio. If the standard error of unit weight of the parallaxes is denoted s_o, the standard error s_{oh} can be found from the expression $s_{oh} = \frac{d}{b}\, s_o$ where d is the distance from the camera base to the intersected point and b is the base. The ratio $\frac{d}{b}$ is usually called the distance-base ratio. For points located much above or below the reference plane, the formula system above means a certain approximation, but for general information about the accuracy to be expected in the elevation determinations, the approximations can be accepted. Moreover, the complete formulas can be derived from the general rotation or coordinate transformation formulas without difficulty.

For the elevation difference between two points M and N, the weight number is (with certain approximations as mentioned above):

$$Q_{\Delta h \Delta h} = 2 + (X_m - X_n)^2 \, Q_{\eta\eta} + (Y_m - Y_n)^2 \, Q_{\xi\xi} + 2(X_m - X_n)(Y_m - Y_n) \, Q_{\eta\xi}$$

and the standard error

$$s_{\Delta h} = s_{oh} \sqrt{Q_{\Delta h \Delta h}}$$

2. Estimation of the Standard Error of Unit Weight and the Error Propagation. According to available information, for instance Nowicki, reference [1], the following data will be used here:

Average diameter of the moon about 3,500 km.

Average distance earth-moon about 390,000 km.

For the angle 16° of libration, the base between the camera stations corresponds to about 102,000 km. Hence, $\frac{d}{b}$ about 3.7

Average scale of available photographs 1:22,000,000 = 1:S
For the standard error of unit weight of parallax measurements s_o, the standard error of unit weight of the elevation measurements is, with minor approximation,

$$s_{oh} = 3.7 \; s_o S$$

For $S = 22 \times 10^6$ is found

$$s_{oh} = 81 \times 10^6 \; s_o$$

The standard error of unit weight of the parallax measurements s_o, which is approximately of the same magnitude as the standard error of unit weight of the image coordinates, is of basic importance, but very few, if any, determinations of this factor seem to have been made concerning moon photographs. Information concerning the geometrical quality of measurements in star photographs has been obtained, and the figures vary between 2 and 5 microns. In moon photographs, the details on the moon are much less defined than star images, and there are also other disturbing factors, such as different shadows in the two photographs taken in two different libration phases. It is very difficult to estimate the standard error of unit weight to be expected, and test measurements and corresponding computations in accord with the parallax condition are highly desirable. From some test measurements for the determination of the precision of image coordinate measurements of the same details as performed by Nathan Fishel, GIMRADA, the standard deviation of one measurement was found to be of the order of magnitude 0.01 mm. This figure is temporarily used as standard error of unit weight of measurements of image coordinates and parallaxes in moon photographs. Hence,

$$s_{oh} = 810 \text{ meters}$$

The error propagation to arbitrary points is then dependent upon the location of the three points which define the reference plane and the actual point according to the formula given above. The lowest standard error is to be expected for $X_i = Y_i = 0$. There we find

$$s_{h\,(0,0)} = 1.15 s_{oh} = 930 \text{ meters}$$

This means that the standard errors of the evaluations under the assumption made can be expected to be of the order of magnitude 1,000 meters, provided that the three points are uniformly distributed, preferably in an equilateral triangle of suitable size.

3. The Coordinates in Planimetry. The differential formulas of the coordinate transformation in planimetry are

$$dx = dx_o + x \frac{db}{b} - y d\alpha$$

$$dy = dy_o + y \frac{db}{b} + x d\alpha$$

Also in this case, the center of gravity (located halfway between the two points to be used for the coordinate transformation) is used as origin of the coordinates. The coordinate errors dx_1, dy_1, dx_2, and dy_2 of the two points are transformed into the elements of the coordinate transformation (absolute orientation) as follows:

$$dx_o = - \frac{dx_1 + dx_2}{2}$$

$$dy_o = -\frac{dy_1 + dy_2}{2}$$

$$\frac{db}{b} = -\frac{X_1 dx_1 + X_2 dx_2 + Y_1 dy_1 + Y_2 dy_2}{X_1^2 + Y_1^2 + X_2^2 + Y_2^2}$$

$$d\alpha = \frac{Y_1 dx_1 + Y_2 dx_2 - X_1 dy_1 - X_2 dy_2}{X_1^2 + Y_1^2 + X_2^2 + Y_2^2}$$

The weight numbers are:

$$Q_{x_o x_o} = Q_{y_o y_o} = \frac{1}{2}$$

$$Q_{bb} = \frac{b^2}{X_1^2 + Y_1^2 + X_2^2 + Y_2^2}$$

$$Q_{\alpha\alpha} = \frac{1}{X_1^2 + Y_1^2 + X_2^2 + Y_2^2}$$

The influence upon an arbitrary point of the errors in the two orientation points can be found from an assumed correction procedure. The final coordinates of an arbitrary point X_i, Y_i can be found from the preliminary coordinates corrected with respect to the coordinate transformation in the two points. In formulas we have

$$X_{final} = X_{prelimin.} + dx_o + X_i \frac{db}{b} - Y_i d\alpha$$

$$Y_{final} = Y_{prelimin.} + dy_o + Y_i \frac{db}{b} + X_i d\alpha$$

Assuming all determinations of the coordinates to be approximately of the same geometrical quality, the weight numbers of the final coordinates are:

$$Q_{X_i X_i} = Q_{Y_i Y_i} = \frac{3}{2} + \frac{X_i^2 + Y_i^2}{[XX] + [YY]}$$

The standard error is then found from

$$s_{X_i} = s_{Y_i} = s_{oc}\sqrt{Q_{X_iX_i}}$$

The standard error of unit weight is in this case assumed to be approximately the same as the standard error of unit weight of the image coordinates enlarged with the scale factor.

Assuming the standard error of unit weight of the image coordinates to be 0.01 mm and the scale factor 22×10^6, the smallest standard error of any point is found for the point of gravity as

$$s_X = s_Y = 270 \text{ meters.}$$

The irregular errors of the coordinates can be expected to be of the order of magnitude (standard error) of 300 meters, provided that the distance between the two points is of reasonable length.

III. THE GEOMETRICAL QUALITY OF LUNAR PHOTOGRAPHY AND MAPPING FROM ACCURACY REQUIREMENTS

4. Requirements of the Lunar Maps According to Assumed Specifications.

Map Scale 1:250,000

Planimetry

Ninety percent of points that are to be depicted on a map shall be located within 800 feet of true position. No points shall be displaced more than 2,000 feet from true position.

$$800 \text{ feet} = 0.305 \cdot 800 = 244 \text{ meters} \qquad (1)$$

Assuming a normal distribution of the errors and a reasonable level, the relation between standard error s and the 90-percent condition 244 meters is

$$1.6449s = 244$$

$$s = \frac{244}{1.6449} = 148 \text{ meters} \qquad (2)$$

This standard error is to be understood as a radial standard error.

Assuming the relation between the radial standard error and the standard errors of the x- and y-coordinates to be $\sqrt{2}:1$ [reference 5], the standard errors of these coordinates are

$$s_x = s_y = 105 \text{ meters} \qquad (3)$$

This standard error is to be understood as a root mean square value of the errors of details on the map in relation to the correct positions in a local coordinate system on the moon. So far, this system is assumed to be determined with the aid of two well-defined photogrammetrically determined points.

Elevations

From the condition that 90-percent of points that are to be indicated on the moon map shall be located within 150 feet of true vertical position, relative to an established lunar datum and again assuming a normal distribution of the elevation discrepancies, the standard error of the elevation is found to be

$$\frac{46}{1.6449} = 28 \text{ meters} \qquad (4)$$

This standard error is to be interpreted as a root mean square value of the discrepancies between the elevations as taken from the map and the given elevations in a local elevation system on the moon. This latter system is assumed to be determined by some of the measured elevations themselves.

Map Scale 1:25,000

Planimetry

The 90-percent value is 80 feet or 24.4 meters. This corresponds to a standard radial error of 14.8 meters and standard errors of the coordinates of

$$s_x = s_y = 10.5 \text{ meters} \qquad (5)$$

Elevations

The 90-percent value is 15 feet or 4.6 meters and the corresponding standard error $s_h = 2.8$ meters. (6)

5. <u>Requirements on Geometrical Quality of the Image Coordinates.</u> According to the derivations made, the standard errors to be expected in planimetry and elevation can, with some generalizations and approximations, be expressed as follows:

$$s_X = s_Y = 1.3 s_o S \tag{7}$$

$$s_h = 1.2 \cdot 3.7 s_o S = 4.44 s_o S \tag{8}$$

s_o is the standard error of unit weight of the image coordinates and of the parallaxes. S is the scale factor of the photographs, determined as $S = \frac{d}{c}$, where d is the distance to the moon and c the camera constant (focal length of the telescope).

<u>Photography for the Scale 1:250,000</u>

Planimetry

From the required geometrical quality (formula 3) and the formula for the quality to be expected from the photogrammetric procedure (formula 7), we find the following relations:

$$1.3 s_o \frac{d}{c} = 105$$

or

$$\frac{s_o}{c} = \frac{105}{1.3d} = \frac{80.7692}{d} \tag{9}$$

d is to be expressed in meters and s_o and c in the same units.

Elevations

From formulas 4 and 8, it is found that:

$$4.44 s_o \frac{d}{c} = 28$$

$$\frac{s_o}{c} = \frac{28}{4.44d} = \frac{6.3063}{d} \tag{10}$$

d is to be expressed in meters and s_o and c in the same units.

Photography for the Scale 1:25,000

Planimetry

From formulas 5 and 7:

$$1.3 s_0 \frac{d}{c} = 10.5$$

$$\frac{s_0}{c} = \frac{8.07692}{d}$$

Elevations

From formulas 6 and 8:

$$\frac{s_0}{c} = \frac{0.63063}{d}$$

d is to be expressed in meters and s_0 and c in the same units.

In all formulas derived, it is assumed that convergent photography is used when the libration is maximum (about 16°).

6. Some Applications of the Formulas. According to available information, the smallest standard error of unit weight of image coordinates and parallaxes to be expected is of the order of magnitude 3 microns.

Using this figure in formulas (9) and (10) and the distance $d = 39 \cdot 10^7$ meters, the following values of the camera constants (focal lengths) of the telescopes would be necessary:

For the map of 1:250,000:

Planimetry

From (9): $c = 14.5$ meters

Elevations

From (10): $c = 186$ meters

The indicated result means that a telescope with a focal length of approximately 186 meters would be necessary for taking the photographs from the earth, if the geometrical quality of the elevations are to have a root mean square value equal to 28 meters. The telescope dimensions and the photogrammetric mapping of the moon from earthbased photography, with the mentioned geometrical quality, appear unrealistic. Therefore, it should be understood that when the geometrical quality is required to maintain any standard, the terrestrial photography intended should be evaluated comprehensively in order to determine whether the desired quality can be obtained.

7. Estimation of the Relative Geometrical Quality in Planimetry and Elevation on the Moon from Stereophotogrammetric Measurements in Photographs Taken with a Telescope of 150-foot Effective Focal Length. With some generalizations and approximations, the standard errors in planimetry and elevation on the moon, after photogrammetric measurements, can be expressed as follows:

$$s_X = s_Y = 1.3\, s_o S$$

$$s_h = 4.44\, s_o S$$

Here

s_o = standard error of unit weight of image coordinates and parallaxes, in the scale of the images.

S = the scale factor of the photographs.

Maximum libration (about 16°) is assumed, and further, it is assumed that the errors of the image coordinates and parallaxes are of irregular nature and are normally distributed at a reasonable level.

The scale factor S can be expressed as $S = \frac{d}{c}$ where d is the distance to the moon and c is the effective focal length (camera constant) of the telescope.

d is of the order of magnitude $39 \cdot 10^7$ meters, and c is assumed to be 150 feet, or about 50 meters.

Hence, the standard errors in planimetry and elevation can be written:

$$s_X = s_Y = \frac{1.3 s_o \cdot 39 \cdot 10^7}{50} = 1.01 \cdot 10^7\, s_o$$

$$s_h = \frac{4.44 s_o \cdot 39 \cdot 10^7}{50} = 3.46 \cdot 10^7 s_o$$

The standard errors are tabulated for different values of s_0. Standard errors in planimetry and elevation for different values of s_0 are:

s_0 (mm)	S_X S_Y (meter)	S_h (meter)
0.001	10	35
0.002	20	70
0.003	30	104*
0.004	40	138
0.005	50	173
0.006	60	208
0.007	70	242
0.008	80	277
0.009	90	311
0.010	100	346
0.015	150	519
0.020	200	692
0.030	300	1038

*This value of s_0 is the lowest to be expected.

From calibration of the telescope and of the photographs under operational conditions, the s_0 value for the instrument is to be determined. From parallax measurements in stereoscopic photographs, which are to be used for the mapping, the actual value of s_0 can be estimated according to the method of least squares. Confidence intervals can be estimated from the number of redundant observations (degrees of freedom) and the confidence level, usually 5 percent.

8. Estimations of the Relative Geometrical Quality from Available Satellite and Balloon Photography. The highest geometrical quality to be expected in lunar mapping from available photographs can be estimated from the formulas derived.

Assuming the scale 1:22,000,000 in the photographs and s_0 = 0.003 mm, the formulas (7) and (8) give the order of magnitude of the standard errors to be expected, as follows:

$$S_x = S_y = 86 \text{ meters} = 282 \text{ feet}$$

$$S_h = 293 \text{ meters} = 960 \text{ feet}$$

For photography from satellites or balloons, similar principles can be used, as shown above. In order to fulfill minimum elevation requirements for lunar mapping, stereoscopic photography from satellites or lunar orbitors appears to be the best method. With reference to the plotting of the photographs, the actual geometrical quality to be expected can be estimated [Section I] from redundant parallax observations and computations. If "vertical" stereoscopic photography is to be used, the basic formulas must be modified.

Finally, it should be emphasized again, if suitable lunar maps are to be made, then calibration of the cameras and tests of the geometrical quality of image coordinates of the lunar photographs must be made under operational conditions with clearly defined requirements for the geometrical quality that must be fulfilled.

REFERENCES

1. Nowicki, Albert L., oral information on moon mapping in general, according to the procedure used at Army Map Service, 1962.

2. Hallert, B., Photogrammetry, McGraw-Hill, New York, 1960.

3. Jonsson, A., "On the Accuracy of Stereo-Plotting of Convergent Aerial Photographs," Photogrammetria, 1960-1961, No. 3.

4. Hallert, B.,"Determination of the Accuracy of Terrestrial Stereophotogrammetric Procedures," Photogrammetric Engineering, March 1955.

5. Thompson, M. M., "A Current View of the National Map Accuracy Standards," Surveying and Mapping, December 1961.

APPENDIX

ELEMENTARY DERIVATION OF FUNDAMENTAL FORMULA SYSTEMS FOR ANALYTICAL PHOTOGRAMMETRY

1. Independent pairs of photographs.

It is temporarily assumed that the normal case of photogrammetry is strictly present, i.e., the camera axes are parallel and orthogonal to the base, Fig. 1. The image coordinates are, for reasons which will be explained later on, written with parentheses (x'), (y') in the left and (x"), (y") in the right photograph. The origins of the image coordinate systems are located in the principal points (H') and (H"), respectively, and the principal distances (camera constants or calibrated focal lengths) are denoted (c_1) and (c_2), respectively. The central projections are temporarily assumed to be mathematically correct, and all other data are assumed to be errorless.

Because the principal distances (c_1) and (c_2) are assumed to be different, the image coordinates of one of the photographs, for instance those of the right photograph, are transformed to a photograph with the principal distance (c_1) as in the left photograph of Fig. 1. The transformation is a simple similarity transformation and is made through multiplying the image coordinates (x"), (y") by the factor

$$\frac{(c_1)}{(c_2)}$$

The transformed coordinates are then transferred to the left photograph as shown in Fig. 1. Then, immediately obtained from similarity is

$$z = \frac{b(c_1)}{(x') - \frac{(x'')\,(c_1)}{(c_2)}} = \frac{(c_1)\,(c_2)}{(x')\,(c_2) - (x'')\,(c_1)}\, b \qquad (1)$$

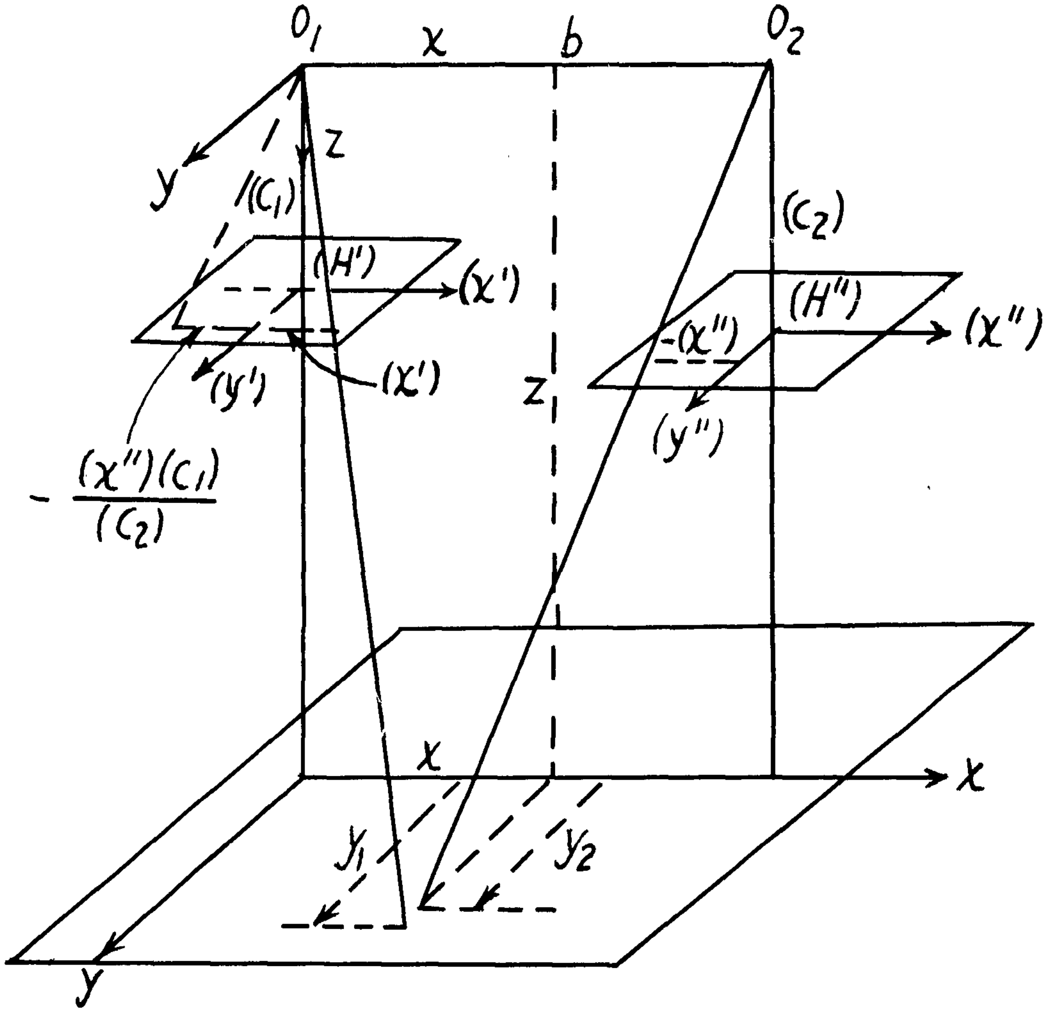

Fig. 1. The normal case. Independent pair of photographs. Derivation of the relation between transformed image coordinates (x'), (y'), (x"), (y") and model coordinates x, y, z. The image coordinates of the right photograph are first transformed to the principal distance (c_1) of the left photograph. Then the fundamental principles of the normal case of stereophotogrammetry are applied. Note that the principal distances (c_1) and (c_2) as well as the positions of the principal points (H') and (H") are functions of the image coordinates and the elements of the interior and relative orientation of the rotated photographs.

$$x = \frac{(x')z}{(c_1)} = \frac{(c_2)\ (x')}{(x')\ (c_2) - (x'')\ (c_1)}\quad b \tag{2}$$

$$y_1 = \frac{(y')z}{(c_1)} = \frac{(c_2)\ (y')}{(x')\ (c_2) - (x'')\ (c_1)}\quad b \tag{3}$$

$$y_2 = \frac{(y'')z}{(c_2)} = \frac{(c_1)\ (y'')}{(x')\ (c_2) - (x'')\ (c_1)}\quad b \tag{4}$$

These coordinates are evidently determined in a local coordinate system, the origin of which is located in the left exposure station O_1. Through coordinate transformation they can be transformed and expressed in another coordinate system (absolute orientation) with the aid of a sufficient number of control points or other control data, at least seven parameters. The condition that corresponding rays must intersect can evidently be written

$$y_1 = y_2 \tag{5}$$

or according to (3) and (4)

$$\frac{(y')}{(c_1)} - \frac{(y'')}{(c_2)} = 0 \tag{6}$$

or

$$(y')\ (c_2) = (y'')\ (c_1) \tag{7}$$

If two rays do not intersect, a y-parallax (vertical parallax according to terrestrial photogrammetry) will appear, defined as

$$p_y = y_1 - y_2 \tag{8}$$

After substitution of the expressions (3) and (4) is found

$$p_y = \left\{ \frac{(y')}{(c_1)} - \frac{(y'')}{(c_2)} \right\} z = \frac{(y')\ (c_2)\ -\ (y'')\ (c_1)}{(x')\ (c_2)\ -\ (x'')\ (c_1)}\ b \tag{9}$$

If the parallax is assumed to be caused by small errors, the following expression is generally used

$$p_y = dy_1 - dy_2$$

1.1 Determination of the relative orientation.

If the elements of the relative orientation of the photographs are not sufficiently known or are completely unknown, the expression (9) can be used for a determination of the elements. There are five elements of the relative orientation, and relating to these elements are assumed conditions requiring five angles(independent pairs of photographs). Both the exposure stations are, in other words, assumed to be fixed.

The five angles are usually chosen: two angles (φ_1 and κ_1) of the left and three angles (ω_2, φ_2, κ_2) of the right photograph. If a photograph in the moment of exposure was rotated through three angles around perpendicular axes through the perspective center (Fig. 2) and the image coordinates x_r, y_r and the principal distance c are given for a point P in the rotated photograph, the corresponding coordinates (x_r), (y_r), (c) of point P can be expressed in the original coordinate system (before the rotation) with the aid of a well-known coordinate transformation through rotation of axes. The actual transformation formulas depend upon the order of the rotations and the sign of the angles. Here those arrangements and signs will be used which are to be found in the Wild Autographs A7 through A9 (ω primary, φ secondary, κ tertiary), but completely arbitrary arrangements and signs can be assumed. The transformation formulas can be derived with different methods, see for instance reference [1] where a very elementary but completely general procedure has been applied.

For the conditions as shown in Figs. 2 and 3, the transformation formulas from the system x_r, y_r, c into the system (x_r), (c) are:

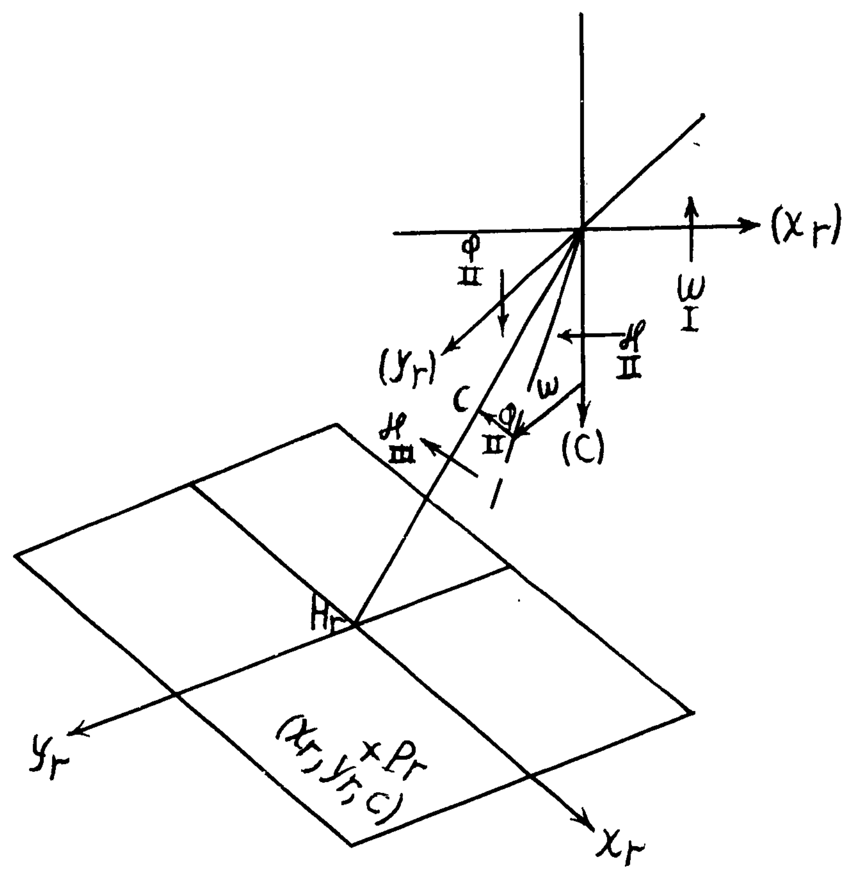

Fig. 2. Transformation of rotated image coordinates x_r, y_r, c to the original system (x_r), (y_r), (c).

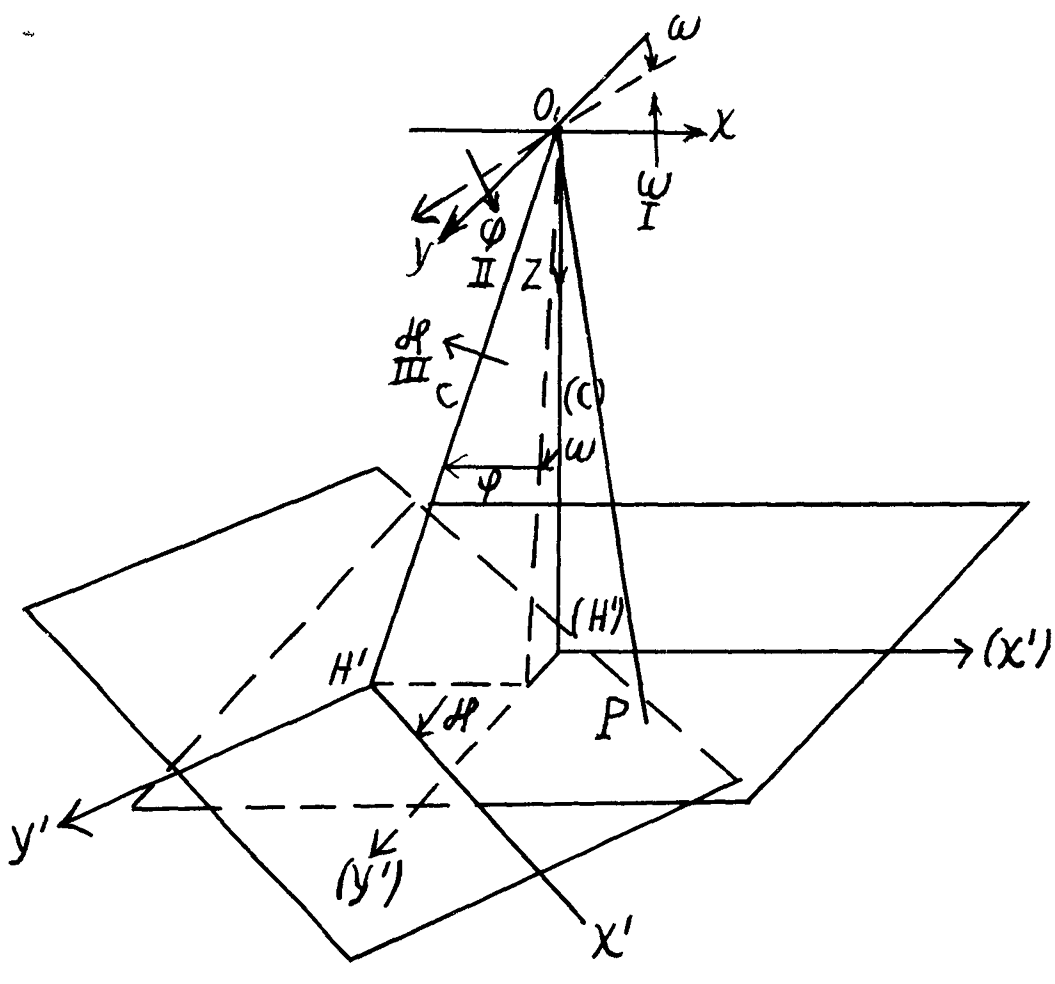

Fig. 3. Transformation of the image coordinates x', y', c of the left photograph to the coordinates (x'), (y'), (c) in the x-, y-, z-system.

$$(x_r) = x_r \cos\varphi\cos\kappa - y_r\cos\varphi\sin\kappa - c\,\sin\varphi \tag{10a}$$

$$(y_r) = x_r\,(\sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa) +$$

$$+ y_r\,(\cos\omega\cos\kappa - \sin\omega\sin\varphi\sin\kappa) + c\,\sin\omega\cos\varphi \tag{10b}$$

$$(c) = x_r\,(-\sin\omega\sin\kappa + \cos\omega\sin\varphi\cos\kappa) -$$

$$- y_r\,(\cos\omega\sin\varphi\sin\kappa + \sin\omega\cos\kappa) + c\,\cos\omega\cos\varphi \tag{10c}$$

It should be noted that the coordinate (c) will vary from point to point and that, consequently, if the transformation is regarded as a transformation of image coordinates from a rotated photograph to a not rotated photograph, this latter one will obtain a specific principal distance (c) for each individual point, Fig. 3.

The expressions (10a) through (10c) are next applied to the two photographs, the angles of rotations of which are φ_1, κ_1 and ω_2, φ_2, κ_2, respectively. ω_1 can be chosen arbitrarily because so far only the relative orientation is considered. It can therefore be chosen zero, and ω_2 is consequently temporarily to be regarded as a difference only.

From measured image coordinates x', y' and x'', y'' and the principal distance c (equal for the two photographs) are obtained

$$(x') = x'\cos\varphi_1\cos\kappa_1 - y'\cos\varphi_1\sin\kappa_1 - c\,\sin\varphi_1 \tag{11a}$$

$$(y') = x'\sin\kappa_1 + y'\cos\kappa_1 \tag{11b}$$

$$(c_1) = x'\sin\varphi_1\cos\kappa_1 - y'\sin\varphi_1\sin\kappa_1 + c\,\cos\varphi_1 \tag{11c}$$

$$(x'') = x''\cos\varphi_2\cos\kappa_2 - y''\cos\varphi_2\sin\kappa_2 - c\,\sin\varphi_2 \tag{12a}$$

$$(y'') = x''(\sin\omega_2\sin\varphi_2\cos\kappa_2 + \cos\omega_2\sin\kappa_2) +$$

$$+ y''(\cos\omega_2\cos\kappa_2 - \sin\omega_2\sin\varphi_2\sin\kappa_2) + c\,\sin\omega_2\cos\varphi_2 \tag{12b}$$

$$(c_2) = x''(-\sin\omega_2\sin\kappa_2 + \cos\omega_2\sin\varphi_2\cos\kappa_2) -$$

$$- y''(\cos\omega_2 \sin\varphi_2 \sin\kappa_2 + \sin\omega_2 \cos\kappa_2) + c \cos\omega_2 \cos\varphi_2 \quad (12c)$$

If the angles of the relative orientation are assumed to be known with sufficient quality, the expressions (11) through (12) can directly be computed for each point and then substituted into the expressions (1) through (4). The conditions (5) through (7) can be applied to every pair of rays and serve as checks that angles, image coordinates, frequently other data, and the calculations are correct.

Because there are always inevitable sources of errors, certain discrepancies must always be expected in the condition of intersection of pairs of rays. Such discrepancies can then be used for the determination of the sources of errors. Usually, and in particular in aerial photogrammetry but sometimes in terrestrial photogrammetry too, the angles used in the first calculation are approximate and are consequently regarded to be affected with errors, which must be determined. First the discrepancy p_y is computed in at least five suitably located points with the expression (9). These discrepancies are then used for the determination of corrections to the approximate angles. Since the functions to be used - the expression (9) after substitution of the expressions (11a) through (12c) - are trigonometrical (transcendent), a direct solution is generally not possible. An iterative procedure must generally be used. First, differential formulas of the complete expression (9) are developed around the approximate values of the angles. Then the necessary five linear equations can be set up and solved, provided that the general conditions for the solution of such an equation system are fulfilled. If the found corrections to the preliminary values of the angles are too large, the procedure is repeated after a differentiation around the corrected values of the angles. The procedure is iterated until the corrections are smaller than certain tolerances, which can be determined with the aid of statistical principles and with respect to the basic accuracy of the procedure. This accuracy is to be determined from redundant observations and with the aid of the method of least squares.

As an example, the derivation of linear differential formulas of the expression (9) will next be shown for the important normal case, i.e., under the assumption that the approximate values of the angles are zero. Under such assumptions, cosines and sines in the expressions (11) through (12) are to be substituted by 1 and the differentials of the angles, respectively. Hence

$$(x') = x' - y'd\kappa_1 - cd\varphi_1 \quad (13a)$$

$$(y') = y' + x'd\kappa_1 \tag{13b}$$

$$(c_1) = c + x'd\varphi_1 \tag{13c}$$

$$(x'') = x'' - y''d\kappa_2 - cd\varphi_2 \tag{14a}$$

$$(y'') = y'' + x''d\kappa_2 + cd\omega_2 \tag{14b}$$

$$(c_2) = c + x''d\varphi_2 - y''d\omega_2 \tag{14c}$$

All squares and products of differentials are neglected.

After substitution of (13) and (14) into (9) is obtained

$$y_1 - y_2 + p_y =$$

$$\frac{y' - y'' + x'd\kappa_1 - \frac{x'y''}{c} d\varphi_1 - x''d\kappa_2 + \frac{x''y'}{c} d\varphi_2 - \left(1 + \frac{y'y''}{c^2}\right) cd\omega_2}{(x'-x'')\left(1 + \frac{-y'd\kappa_1 - \left(1 + \frac{x'x''}{c^2}\right) cd\varphi_1 + y''d\kappa_2 + \left(1 + \frac{x'x''}{c^2}\right) cd\varphi_2 - \frac{x'y''}{c} d\omega_2}{x'' - x''}\right)} b \tag{15}$$

Since the fraction in the denominator under normal circumstances is smaller than 1, the entire expression can be developed into a geometrical series, according to the well-known expression

$$S = \frac{a}{1 - q} = a + aq + aq^2 + \ldots\ldots. \qquad (q < 1)$$

Terms containing differentials of second or higher order are neglected. After simple developments and with minor approximations, the following well-known y-parallax formula is found from (15)

$$p_y \frac{x'-x''}{b} = p_y' = x'd\kappa_1 - \frac{x'y''}{c} d\varphi_1 - x''d\kappa_2 + \frac{x''y'}{c} d\varphi_2 - \left(1 + \frac{y'y''}{c^2}\right) cd\omega_2 \tag{16}$$

It should be noted that $x'-x''$ approximately is the base b' in the photographs for limited elevation (depth) differences in the object and that, consequently, the fraction $\frac{b'}{b}$ approximately represents the scale $\frac{c}{z} = \frac{1}{S}$ of the photographs. Further should be noted that p_y in (15) represents that part of the total y-parallax, which is caused by the approximation of the angles only and that it should be defined as $p_y = y_1 - y_2$. The expression $p_y \frac{x'-x''}{b}$ in (16) is the parallax according to (9), transformed to the image scale where it is denoted p'_y.

For the approximate normal case and limited elevation (depth) differences, certain simplifications can be introduced in (16). Assuming $y'=y''$ and $x''= x'- b'$ is found

$$y'-y'' = p'_y = x'd\kappa_1 - \frac{x'y'}{c} d\varphi_1 - (x'- b')d\kappa_2 + \frac{(x'-b')\ y'}{c} d\varphi_2 - (1+\frac{y'^2}{c^2})cd\omega_2 \tag{17}$$

These differential formulas are very well known in photogrammetry and must be regarded as the most important mathematical expressions for the important normal case in practice. The final values of the elements of the relative orientation are to be determined indirectly from the basic observations of image coordinates, with the aid of the elements of the interior orientation and using the condition that corresponding rays shall intersect. This procedure is identical with the practical procedure of the relative orientation in a stereoscopic restitution instrument. The accuracy of the procedure can uniquely be determined from redundant observations according to the method of least squares, and tolerances for the residuals of the elements of the relative orientation and other data can be determined accordingly. The elements of the interior orientation, including possible regular errors of the central projection (image coordinates) and the irregular errors of the image coordinates are evidently of basic importance for the accuracy of the determination of the elements of the relative orientation and all functions of these elements. The influence of errors of the elements of the interior orientation upon the elements of the relative orientation can, for instance, be found from differentiations of the complete analytical expressions (11), (12), (9) and then application of the laws of error propagation. It should be noted that the image coordinates x',y' and x'',y'' in the expressions (11a) through (12c) in principle are <u>differences</u> between measured image coordinates (subscript m) and the coordinates of the principal points (subscript o), or in formulas

$$x' = x'_m - x'_o \qquad\qquad x'' = x''_m - x''_o$$

$$y' = y'_m - y'_o \qquad\qquad y'' = y''_m - y''_o$$

Consequently, the propagation of errors in the coordinates of the principal points can be studied from these expressions and differentiations. It must be emphasized that the errors of the principal distance and of the principal points as well as of other elements of the interior orientation (distortions, etc.) must be determined in connection with the calibration of the camera and of the photographs, which must be performed under real operational conditions, in addition to laboratory tests. Tolerances must also be specified.

The relative orientation and the condition that corresponding pairs of rays shall intersect, (5) through (9), is evidently a very valuable check of certain regular errors of the elements of the interior orientation and also makes possible a determination of certain regular non-projective errors, which are not correlated between corresponding points in the two photographs. The residual irregular discrepancies of the intersection condition can finally be estimated as a standard error of unit weight and can frequently be used for investigations of the error propagation in functions of the elements of the relative orientation, i.e., the results of the entire photogrammetric procedure.

1.2 Determination of model coordinates.

After the determination of the elements of the relative orientation according to the iterative procedure as described above, the coordinates z, x, and y can be computed on a temporary scale according to (1) through (4). These coordinates are termed model coordinates. Because the two y-coordinates y_1 and y_2 generally are somewhat different due to residual y-parallaxes, the average of the two values is used as the final y-coordinate, i.e.

$$y = \frac{y_1 + y_2}{2} \tag{18}$$

It would be more correct to determine weights of y_1 and y_2 according to (3) and (4) and then to determine the weighted average.

For many purposes, the differential formulas of (1) through (4) are of great interest. The formulas can be derived in the same manner as was shown above concerning the y-parallaxes.

Hence

$$dz = z\frac{y'}{(x'-x'')}\,d\varkappa_1 + \left(1+\frac{x'^2}{c^2}\right)\frac{c}{(x'-x'')}\,z\,d\varphi_1 - z\frac{y''}{(x'-x'')}\,d\varkappa_2 -$$
$$- \left(1+\frac{x''^2}{c^2}\right)\frac{c}{(x'-x'')}\,z\,d\varphi_2 + \frac{x''y''}{(x'-x'')c}\,z\,d\omega_2 \tag{19}$$

It should be noted that z is positive in the direction of the photography. The elevation error dh in a model of approximate vertical photographs must therefore have reversed sign in comparison with (19) if the elevations are positive upwards.

Further are found

$$dx = z\frac{x''y'}{(x'-x'')\;c}\,d\varkappa_1 + \left(1+\frac{x'^2}{c^2}\right)\frac{x''}{(x'-x'')}\,z\,d\varphi_1 - z\frac{x'y''}{(x'-x'')\;c}\,d\varkappa_2 -$$
$$- \left(1+\frac{x''^2}{c^2}\right)\frac{x'}{(x'-x'')}\,z\,d\varphi_2 - \frac{x'x''y''}{(x'-x'')c^2}\,z\,d\omega_2 \tag{20}$$

$$dy_1 = \left(x'+\frac{y'^2}{x'-x''}\right)\frac{z}{c}\,d\varkappa_1 + \left(1+\frac{x'x''}{c^2}\right)\frac{y'z}{x'-x''}\,d\varphi_1 - \frac{y'y''z}{(x'-x'')c}\,d\varkappa_2 -$$
$$- \left(1+\frac{x''^2}{c^2}\right)\frac{y'z}{x'-x''}\,d\varphi_2 + \frac{x''y'y''z}{(x'-x'')c^2}\,d\omega_2 \tag{21}$$

$$dy_2 = \frac{y'y''z}{(x'-x'')c}\,d\varkappa_1 + \left(1+\frac{x'^2}{c^2}\right)\frac{y''z}{x'-x''}\,d\varphi_1 - \left(\frac{y''^2}{x'-x''} - x''\right)\frac{z}{c}\,d\varkappa_2 -$$
$$- \left(1+\frac{x'x''}{c^2}\right)\frac{y''z}{x'-x''}\,d\varphi_2 + \left(1+\frac{x'y''^2}{(x'-x'')\;c^2}\right)z\,d\omega_2 \tag{22}$$

If the final y-coordinate is determined according to (18), the corresponding differential formula is

$$dy = \frac{dy_1 + dy_2}{2} \tag{23}$$

2. Dependent pairs of photographs (extension).

In this case, the elements of the relative orientation of one (usually the right) photograph only are used. Consequently, the two translations by and bz must also be used together with the angles ω_2, φ_2, and κ_2.

According to Fig. 4, the base b is divided into three components bx, by, and bz. The directions and the arrangement of the translations are chosen according to the Wild Autographs.

Directly from Fig. 4, the model coordinates can be determined from similarity. The base component bx must be corrected with the term $- \frac{(x'')bz_2}{(c_2)}$, which is also derived from similarity.

The following expressions are found

$$z = \frac{bx - \dfrac{(x'')bz_2}{(c_2)}}{x' - \dfrac{(x'')c_1}{(c_2)}} c_1 = \frac{(c_2)bx - (x'')bz_2}{(c_2)x' - (x'')\ c_1} c_1 \tag{30}$$

$$x = \frac{x'z}{c_1} = \frac{(c_2)bx - (x'')bz_2}{(c_2)x' - (x'')c_1} x' \tag{31}$$

$$y_1 = \frac{y'z}{c_1} = \frac{(c_2)bx - (x'')bz_2}{(c_2)x' - (x'')\ c_1} y' \tag{32}$$

$$y_2 = \frac{(y'')(z - bz_2)}{(c_2)} - by_2 = \frac{c_1 bx - x'bz_2}{(c_2)x' - (x'')c_1} (y'') - by_2 \tag{33}$$

The expressions for (x''), (y''), and (c_2) are given in (12a) through (12c). If the same camera was used for taking both photographs, $c_1 = c$.

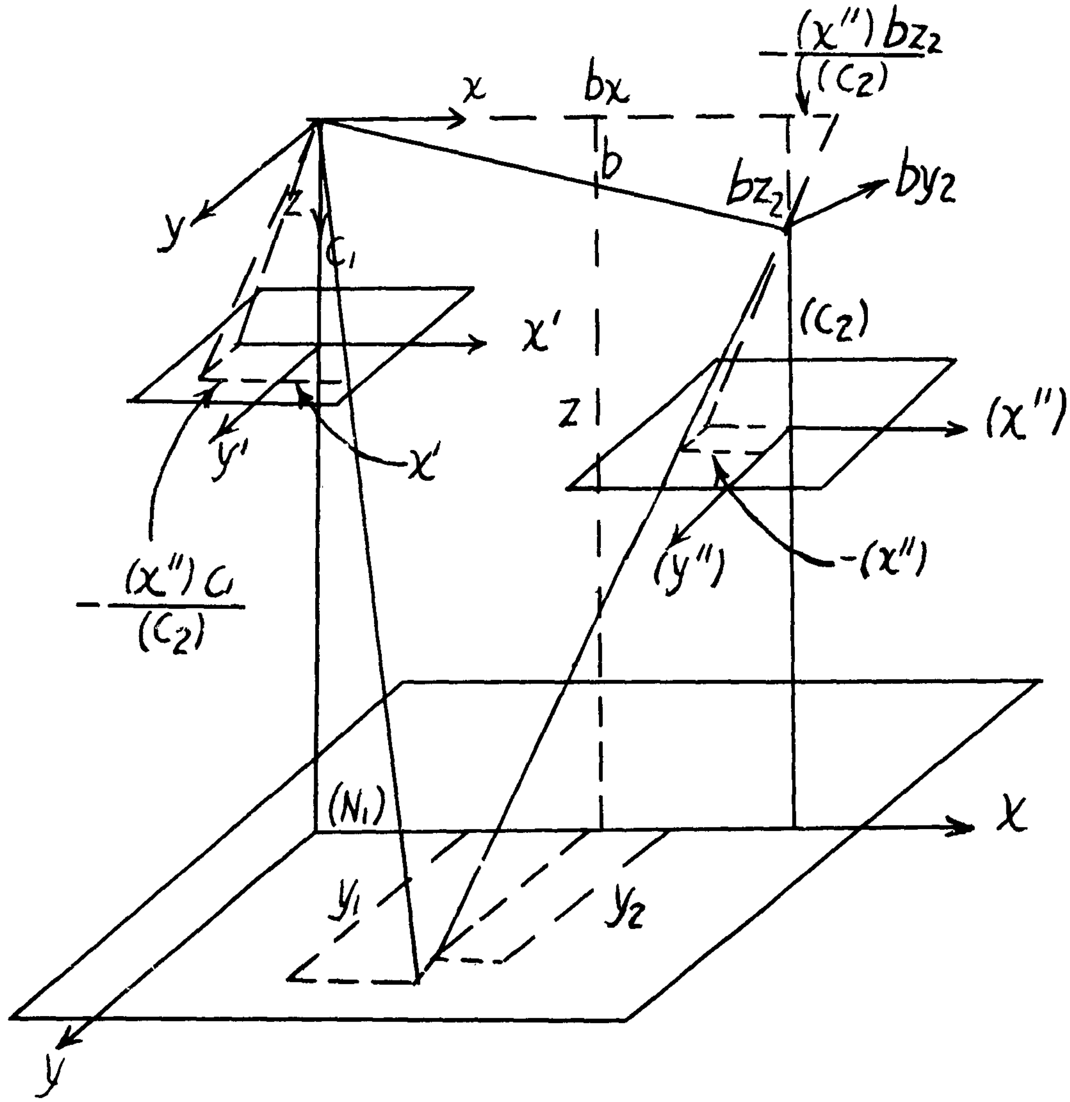

Fig. 4. The normal case. Extension. The influence of the base component bz_2.

Further is found

$$y_1 - y_2 = p_y = \frac{y'z}{c_1} - \frac{(y'')(z-bz_2)}{(c_2)} + by_2 \tag{34}$$

After substitution of the expressions (32) and (33) is found

$$p_y = \frac{bx\left\{(c_2)y'-(y'')c_1\right\} + bz_2\left\{(y'')x'-(x'')y'\right\} + by_2\left\{(c_2)x'-(x'')c_1\right\}}{(c_2)x' - (x'')c_1} \tag{35}$$

Differential formulas can be derived as shown above, see (15) through (17). For the approximate normal case and limited elevation (depth) differences in the object is found

$$\frac{c}{z} p_y = p_y' = -x'' d\varkappa_2 + \frac{x''y'}{c} d\varphi_2 - \left(1 + \frac{y'^2}{c^2}\right) c d\omega_2 + \frac{y'}{z} dbz_2 + \frac{c}{z} dby_2 \tag{36}$$

Further are

$$dz = - \frac{y''}{x'-x''} z d\varkappa_2 + \left(x'' - \frac{c^2+x'x''}{x'-x''}\right) \frac{z}{c} d\varphi_2 - \frac{x''}{x'-x''} dbz_2 + \frac{x''y''}{(x'-x'')} \frac{z}{c} d\omega_2 \tag{37}$$

Note that $dz = -dh$ for approximately vertical aerial photographs.

$$dx = - \frac{x'y''}{(x'-x'')} \frac{z}{c} d\varkappa_2 - \frac{(x''^2 + c^2)b}{(x'-x'')^2 c} x' d\varphi_2 - \frac{x'x''}{(x'-x'')c} dbz_2 + \frac{x'x''y''z}{(x'-x'')c^2} d\omega_2 \tag{38}$$

3. The absolute orientation.

The model coordinates can be transformed to the coordinate system of the object with the aid of one scale change, three translations, and three rotations. For this purpose, at least seven parameters must be given, for instance, two completely determined control points in the coordinate system of the object and one additional coordinate (in terrain mapping, one elevation point).

Certain conditions concerning the choice and location of the control data must be noted.

It should be noted that the model coordinate errors due to various sources, for instance, the elements of the relative orientation, will become compensated in the control points but that the elements of the absolute orientation will become affected with errors accordingly. Therefore, the errors will show up in other transformed model points together with the original model errors.

In case of redundant control data, a certain principle must be used for the distribution of the discrepancies. The method of least squares is also, in this case, of importance for a unique solution of the problem under well-defined conditions for the error propagation and usually leads to simple calculations.

Reference

1. Hallert, B.: Photogrammetry. McGraw-Hill. New York 1960.

1- 3-4- -48,617—ARMY—FORT BELVOIR, VA.

AD Accession No.

U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency, Fort Belvoir, Virginia - GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS - K. Bertil P. Hallert

Research Note No. 9, 18 September 1962, 34 pp, 4 illus
DA Task 8T35-12-001-02 Unclassified Report

At present, the preparation of lunar maps requires the aid of photogrammetric methods. For such mapping as well as for all other kinds of mapping, it is most important to determine the geometrical quality to be expected in planimetry and elevation. Topographic mapping of the earth can be checked with ordinary geodetic methods, but for obvious reasons, the geometrical quality of lunar mapping can be checked in a similar manner only after man has landed on the moon. Therefore, the only possibility is to determine the basic accuracy of the photographic material and the operations to be used for the mapping and then apply the laws of error propagation. In this paper, an attempt has been made to apply ordinary photogrammetric theory of errors and statistical methods for the determination of the final geometrical quality to be expected. Since no information is available concerning the accuracy of the image coordinates of the lunar photographs, the condition of the intersection of reconstructed rays has been used in a manner similar to that which has been successfully applied for the determination of the geometrical quality of ordinary aerial photogrammetry. The derived formula systems and the procedures used are of an approximate character but give information about the quality to be expected under different assumptions concerning the basic geometrical data.

UNCLASSIFIED

1. Mapping, Charting, and Geodesy - Mapping and Geodetic Research

AD Accession No.

U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency, Fort Belvoir, Virginia - GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS - K. Bertil P. Hallert

Research Note No. 9, 18 September 1962, 34 pp, 4 illus
DA Task 8T35-12-001-02 Unclassified Report

At present, the preparation of lunar maps requires the aid of photogrammetric methods. For such mapping as well as for all other kinds of mapping, it is most important to determine the geometrical quality to be expected in planimetry and elevation. Topographic mapping of the earth can be checked with ordinary geodetic methods, but for obvious reasons, the geometrical quality of lunar mapping can be checked in a similar manner only after man has landed on the moon. Therefore, the only possibility is to determine the basic accuracy of the photographic material and the operations to be used for the mapping and then apply the laws of error propagation. In this paper, an attempt has been made to apply ordinary photogrammetric theory of errors and statistical methods for the determination of the final geometrical quality to be expected. Since no information is available concerning the accuracy of the image coordinates of the lunar photographs, the condition of the intersection of reconstructed rays has been used in a manner similar to that which has been successfully applied for the determination of the geometrical quality of ordinary aerial photogrammetry. The derived formula systems and the procedures used are of an approximate character but give information about the quality to be expected under different assumptions concerning the basic geometrical data.

UNCLASSIFIED

1. Mapping, Charting, and Geodesy - Mapping and Geodetic Research

AD Accession No.

U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency, Fort Belvoir, Virginia - GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS - K. Bertil P. Hallert

Research Note No. 9, 18 September 1962, 34 pp, 4 illus
DA Task 8T35-12-001-02 Unclassified Report

At present, the preparation of lunar maps requires the aid of photogrammetric methods. For such mapping as well as for all other kinds of mapping, it is most important to determine the geometrical quality to be expected in planimetry and elevation. Topographic mapping of the earth can be checked with ordinary geodetic methods, but for obvious reasons, the geometrical quality of lunar mapping can be checked in a similar manner only after man has landed on the moon. Therefore, the only possibility is to determine the basic accuracy of the photographic material and the operations to be used for the mapping and then apply the laws of error propagation. In this paper, an attempt has been made to apply ordinary photogrammetric theory of errors and statistical methods for the determination of the final geometrical quality to be expected. Since no information is available concerning the accuracy of the image coordinates of the lunar photographs, the condition of the intersection of reconstructed rays has been used in a manner similar to that which has been successfully applied for the determination of the geometrical quality of ordinary aerial photogrammetry. The derived formula systems and the procedures used are of an approximate character but give information about the quality to be expected under different assumptions concerning the basic geometrical data.

UNCLASSIFIED

1. Mapping, Charting, and Geodesy - Mapping and Geodetic Research

AD Accession No.

U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency, Fort Belvoir, Virginia - GEOMETRICAL QUALITY OF LUNAR MAPPING BY PHOTOGRAMMETRIC METHODS - K. Bertil P. Hallert

Research Note No. 9, 18 September 1962, 34 pp, 4 illus
DA Task 8T35-12-001-02 Unclassified Report

At present, the preparation of lunar maps requires the aid of photogrammetric methods. For such mapping as well as for all other kinds of mapping, it is most important to determine the geometrical quality to be expected in planimetry and elevation. Topographic mapping of the earth can be checked with ordinary geodetic methods, but for obvious reasons, the geometrical quality of lunar mapping can be checked in a similar manner only after man has landed on the moon. Therefore, the only possibility is to determine the basic accuracy of the photographic material and the operations to be used for the mapping and then apply the laws of error propagation. In this paper, an attempt has been made to apply ordinary photogrammetric theory of errors and statistical methods for the determination of the final geometrical quality to be expected. Since no information is available concerning the accuracy of the image coordinates of the lunar photographs, the condition of the intersection of reconstructed rays has been used in a manner similar to that which has been successfully applied for the determination of the geometrical quality of ordinary aerial photogrammetry. The derived formula systems and the procedures used are of an approximate character but give information about the quality to be expected under different assumptions concerning the basic geometrical data.

UNCLASSIFIED

1. Mapping, Charting, and Geodesy - Mapping and Geodetic Research